BAULKHAM HILLS HIGH SCHOOL
MARKING COVER SHEET

YEAR 12 ADVANCED MATHEMATICS ASSESSMENT JUNE 2010

STUDENT'S NAME:___________________
TEACHER'S NAME:___________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
TOTAL	
PERCENTAGE	

YEAR 12 ADVANCED MATHEMATICS ASSESSMENT JUNE 2010

TIME : 35 MINUTES

NAME	**TEACHER**

DIRECTIONS	▪ Full working should be shown in every question. • Marks may be deducted for careless or badly arranged work. ▪ Use black or blue pen only (*not pencils*) to write your solutions. ▪ No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.	
QUESTION 1.	If $\log_a b = 0.12$ and $\log_a c = 0.23$ find $\log_a \frac{ab}{c^2}$.	2
QUESTION 2.	Differentiate xe^{2x}	2
QUESTION 3.	Differentiate $\frac{\log_e x}{x}$	2
QUESTION 4.	(i) Differentiate e^{x^2} (ii) Hence evaluate $\int_0^1 xe^{x^2}\,dx$	1 2
QUESTION 5.	A particle moves in a straight line such that its distance *x*, in metres , from a fixed point *O* is given by $x = 1 - 2\sin 2t$ where *t* is the time, measured in seconds, commencing at $t = 0$. (i) What is the initial position of the particle? (ii) At what time, and where, does the particle first come to rest? (iii) What is the exact acceleration of the particle when $t = \frac{\pi}{6}$ seconds?	 1 3 2
QUESTION 6.	A particle P is initially at the origin and moves so that its velocity is given by $v = \frac{1}{t+3}$ for $t \geq 0$. (i) Find the acceleration of P when $t = 3$. (ii) What is the exact displacement *x* of P from the origin when $t = 2$?	 2 3

QUESTION 7.	The population of an organism at time *t* is given by $P = Ne^{0.2t}$ where *t* is in days and *N* is a constant. (i) Show that the population increases at a rate proportional to the number present. (ii) When $t = 4$ the population was estimated to be 1.2×10^5. Find *N* to the nearest thousand. (iii) Find, to 2 decimal places, the number of days until the population doubles.	 **2** **2** **2**
QUESTION 8.	A person invests \$5000 at 9% per annum compound interest, compounded monthly. Calculate, to the nearest cent, the total interest earned after 5 years.	**2**
QUESTION 9.	A person is to invest \$1000 at the start of each year into a superannuation fund where the compound interest rate is expected to be 10% per annum. The first \$1000 is invested at the beginning of 2011 and the last is to be invested at the beginning of 2040. Calculate, to the nearest dollar, (i) The amount to which the 2011 investment will have grown by the beginning of 2041. (ii) The amount to which the total investment will have grown by the beginning of 2041.	 **1** **3**
QUESTION 10.	The region beneath the curve $y = e^x + e^{-x}$ which is above the *x*-axis and between the lines $x = 0$ and $x = 1$ is rotated about the *x*-axis. Find the volume of the resulting solid of revolution.	**3**
QUESTION 11.	A loan of \$40000 is to be repaid by equal annual instalments. Compound interest at the rate of 8% p. a. is calculated yearly. If the annual instalment of \$P is made immediately after the interest is added: (i) Show that the amount owing after 2 years is $\$40000 \times 1.08^2 - P(1 + 1.08)$ (ii) Write a similar expression for the amount owing after *n* years. (iii) Find the simplest expression for P if the loan and interest is exactly repaid in *n* years.	 **1** **2** **3**

THE END

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad if \;\; n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \qquad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \; a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Q1. $\log_a \frac{ab}{c^2} = \log_a a + \log_a b - \log_a c^2$

$= \log_a a + \log_a b - 2\log_a c$ -1

$= 1 + 0.12 - 2\times 0.23$

$= 0.66$ -1 (2)

Q2 If $y = xe^{2x}$

$\frac{dy}{dx} = e^{2x}.1 + x.2e^{2x}$ -1

$= e^{2x} + 2xe^{2x}$ -1

or $= e^{2x}(1+2x)$ (2)

Q3 If $y = \frac{\log x}{x}$

$\frac{dy}{dx} = \frac{x.\frac{1}{x} - \log x.1}{x^2}$ -1

$= \frac{1-\log x}{x^2}$ -1 (2)

Q4 (i) If $y = e^{x^2}$

$\frac{dy}{dx} = 2xe^{x^2}$ -1

(ii) $I = \int_0^1 xe^{x^2}dx$

$= \frac{1}{2}\left[e^{x^2}\right]_0^1$ -1

$= \frac{1}{2}(e^1 - e^0)$

$= \frac{1}{2}(e-1)$ -1 (3)

Q5 (i) $x = 1 - 2\sin 2t$

when $t = 0$

$x = 1 - 2\sin 0$

$x = 1$ -1

(ii) $v = -4\cos 2t$

when $v = 0$

$\cos 2t = 0$

$2t = \frac{\pi}{2}$ first

$t = \frac{\pi}{4}$ s

$\therefore x = 1 - 2\sin\frac{\pi}{2}$

$= 1 - 2$

$x = -1$ m -1

(iii) $a = 8\sin 2t$ -1

when $t = \frac{\pi}{6}$

$a = 8\sin\frac{\pi}{3}$

$= 8\times\frac{\sqrt{3}}{2}$

$= 4\sqrt{3}$ m/s² -1

Q6 (i) If $v = \frac{1}{t+3}$

$a = \frac{dv}{dt} = \frac{-1}{(t+3)^2}$ -1

when $t = 3$,

$a = -\frac{1}{6^2} = \frac{-1}{36}$ -1

(ii) If $\frac{dx}{dt} = \frac{1}{t+3}$

then $x = \log(t+3) + c$ -1

when $t = 0$, $x = 0$ $\therefore c = -\log 3$ -1

$\therefore x = \log(t+3) - \log 3$

when $t = 2$,

$x = \log 5 - \log 3$
$= \log\frac{5}{3}$ } -1 (5)

Q7 (i) $P = Ne^{0.2t}$

$\frac{dP}{dt} = 0.2Ne^{0.2t}$ -1

$= 0.2P$

$\therefore \frac{dP}{dt} \propto P$ -1

(ii) when $t = 4$, $P = 1.2\times 10^5$

$\therefore 1.2\times 10^5 = Ne^{0.8}$

$N = \frac{1.2\times 10^5}{e^{0.8}}$ -1

$= 53919$

$\doteqdot 54000$ -1

(iii) $2N = Ne^{0.2t}$

$2 = e^{0.2t}$ -1

$0.2t = \log 2$

$t = 5\ln 2$

$= 3.47$ (2 d.p.) -1 (6)

Q8 $P = 5000$, $N = 60$, $r = \frac{9}{12} = 0.75$

$A_N = P\left(1 + \frac{r}{100}\right)^N$

$A_{60} = 5000\times 1.0075^{60}$ -1

$\therefore$ Int $= 5000\times 1.0075^{60} - 5000$

$= \$2828.41$ -1 (2)

Q9. (i) $A_{30} = 1000 \times 1.1^{30}$

$= \$17449.40$ —1

(ii) Total $= A_{30} + A_{29} + \cdots + A_1$

$= 1000 \times 1.1^{30} + 1000 \times 1.1^{29} + \cdots + 1000 \times 1.1$ —1

G.S. where $a = 1000 \times 1.1$,

$r = 1.1$,

$n = 30$ —1

$\therefore$ Total $= \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{1000 \times 1.1(1.1^{30} - 1)}{1.1 - 1}$

$= \$180943.43$ —1 (4)

Q10. $V = \pi \int_a^b y^2 \, dx$

$= \pi \int_0^1 (e^x + e^{-x})^2 \, dx$

$= \pi \int_0^1 e^{2x} + 2 + e^{-2x} \, dx$ —1

$= \pi \left[\frac{1}{2} e^{2x} + 2x + -\frac{1}{2} e^{-2x} \right]_0^1$

$= \pi \left[\left(\frac{1}{2} e^2 + 2 - \frac{1}{2} e^{-2} \right) - \left(\frac{1}{2} + 0 - \frac{1}{2} \right) \right]$ —1

$= \frac{\pi}{2} (e^2 + 4 - e^{-2}) \, u^3$ —1 (3)

Q11 (i) Amount owing after 1 year

$A_1 = 40000 \times 1.08 - P$

$A_2 = A_1 \times 1.08 - P$ —1

$= (40000 \times 1.08 - P) \times 1.08 - P$

$= 40000 \times 1.08^2 - P(1 + 1.08)$

(ii) $A_N = 40000 \times 1.08^N - P(1 + 1.08 + \cdots + 1.08^{N-1})$

(iii) If $A_N = 0$

$P = \dfrac{40000 \times 1.08^N}{1 + 1.08 + \cdots + 1.08^{N-1}}$ —1

Den. is a G.S. where $a = 1, r = 1.08, n = N$

$\therefore P = \dfrac{40000 \times 1.08^N}{\dfrac{1(1.08^N - 1)}{1.08 - 1}}$ —1

$= \dfrac{0.08 \times 40000 \times 1.08^N}{1.08^N - 1}$